CAN-FORMED PROP BLADES

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INTRODUCTION

While carving props for rubber-power models may be character building, many prefer the simpler method of steaming sheet-balsa blades over a form. The form may be simply a tin can, or other cylindrical object. By selecting the right size can, and by laying the blade blank at the proper angle on the can, one can achieve a remarkably good approximation to a uniform-pitch prop, and one can obtain any desired pitch and camber.

One can certainly select the can size and helix angle by trial and error methods, and certainly dozens of perfectly satisfactory props have been made that way. I think the job is much easier, and the results more certain, when the graphs presented in this paper are used.

Max Chernoff presented an analysis of cylindrical props in the 1964-65 Model Aeronautic Year Book* for props having blades that extend to the propshaft. This paper extends that analysis to the more general case where the blades start at some distance from the propshaft and are supported on "arms," or extensions of the hub.

There are three problems to be considered: a) Selecting the twist rate to give a good approximation to a uniform-pitch blade; b) Selecting the can size and helix angle to give the desired twist rate and the desired camber at the same time; c) Setting the blades on the hub at the correct angle.

All of the mathematical development is tucked away in the appendix where it will do no harm. I will not be offended if you skip the rest of this and jump directly to the "Example." If you follow through the method to your own prop will be a cinch.

DETERMINING THE TWIST RATE

I'll refer to a prop having blades that are bent around a can as a "cylindrical prop;" seems more dignified than talking about tin cans. The blades of such a prop twist at a uniform rate — so many degrees per inch. Figure 1 shows how blade angle varies along the blade; a cylindrical prop is represented by a straight line, while a uniform-pitch prop is represented by a curved line. On all modern indoor props, and on many outdoor props too, the inner parts of the blades are replaced by extensions of the prop hub; the blade starts some 20 to 30% of the way from the shaft to the tip. To minimize the blade-angle error — the difference in blade angle between the cylindrical and uniform pitch prop — we make the errors at the inner end of the blade, the blade tip, and a point near the middle of the blade equal. Note that the tip and hub ends are washed out, while the middle part is washed in.

*Available from NFFS Plans and Publications; P.O. Box 322, Dallas, Oregon 97338; $5.00, plus 50¢ 3rd class or $1.50 airmail postage.

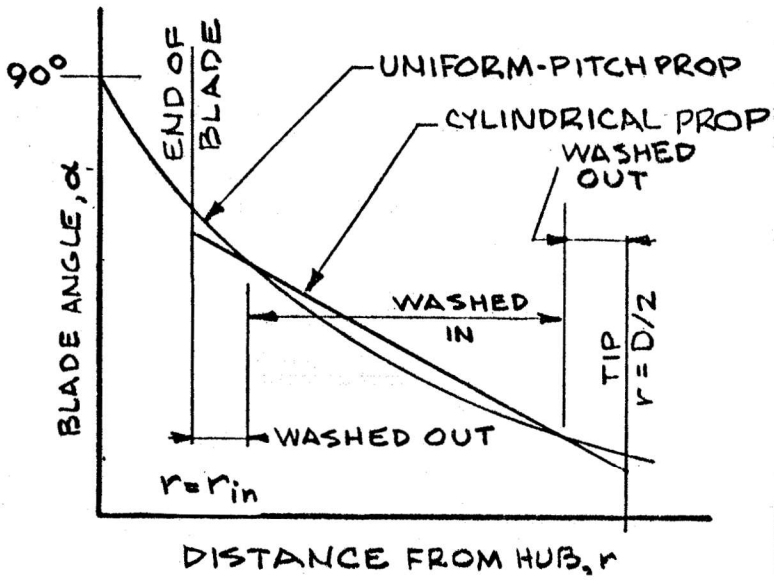


Figure 1: Variation of Blade Angle with Radius

Figure 2 has been constructed for props having blades that start at 20% and 30% of the distance from the shaft to the tip; designated "$r_{in}/\frac{1}{2}D = 0.2$" and "$r_{in}/\frac{1}{2}D = .3$", respectively, on the graphs. The lower part of Figure 2 shows the twist rate that results in minimum blade-angle

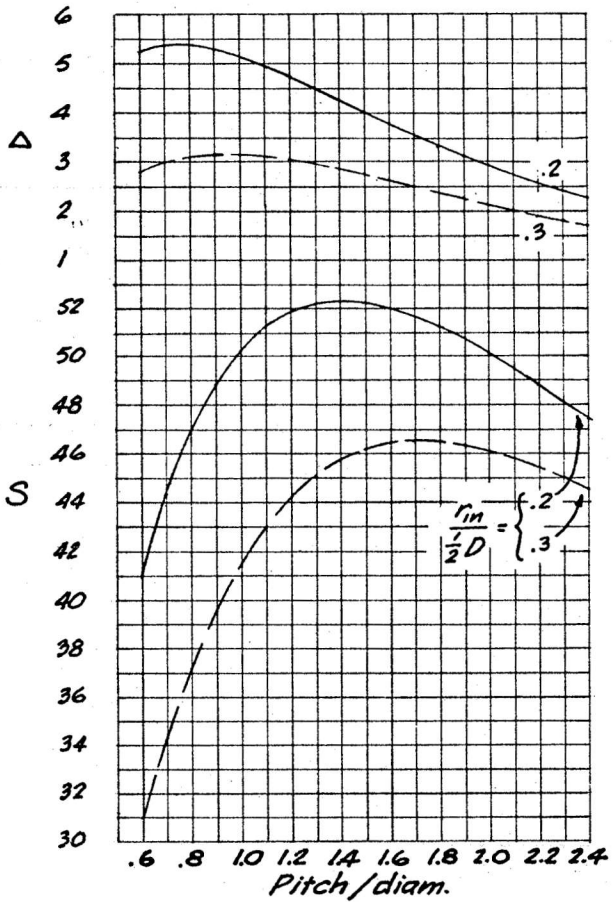


Figure 2: Twist Rate

error for various pitch-diameter ratios. The twist rate, S, represents the total twist that would occur if the blade extended clear to the shaft — merely a convenience. The top part of the graph shows the corresponding blade-angle error, Δ. Note that for a blade that starts 30% of the way out, the maximum angle error is only about 3 degrees.

SELECTING THE CAN SIZE AND HELIX ANGLE

Having selected the twist rate, we must choose a can that gives that twist rate and, at the same time, the desired camber. The blade width enters into the problem too. We express the camber in terms of the ratio of the height of the arch of the bottom of the blade to the chord; h/c. We express the blade width as a fraction of the prop diameter, c/D. The parameter "F" is simply the ratio of the two ratios; (h/c)/(c/D). Along the bottom of Figure 3 the twist rate S is shown. Three sets of curves are shown; each line in each set is for a particular value of F. The lower set of curves gives the ratio of the can diameter to the prop diameter; 2R/D. The middle set gives the helix angle; the angle at which the blade blank is laid along the can, which we call θ. The upper set of curves expresses the amount of "hook" or "arch" in the blades; we needn't be too concerned about that, but props having a value of H/D greater than about 0.015 look a little grotesque.

SETTING THE BLADES AT THE PROPER ANGLE

This is a cinch for anyone that has built props before.

Figure 4 shows the blade angle at a point 80% of the distance from the shaft to the tip.

AN EXAMPLE

Designing your own cylindrical prop will be easy if you follow this example for a Pennyplane prop.

Prop diameter, D = 17 in.
Pitch, P = 25 in.
Blade chord, c = 2 in.
Camber ratio, h/c = 0.11 (11%)
P/D = 25/17 = 1.47
c/D = 2/17 = 0.118
F = (h/c)/(c/D) = 0.11/0.118 = 0.93, say 0.9

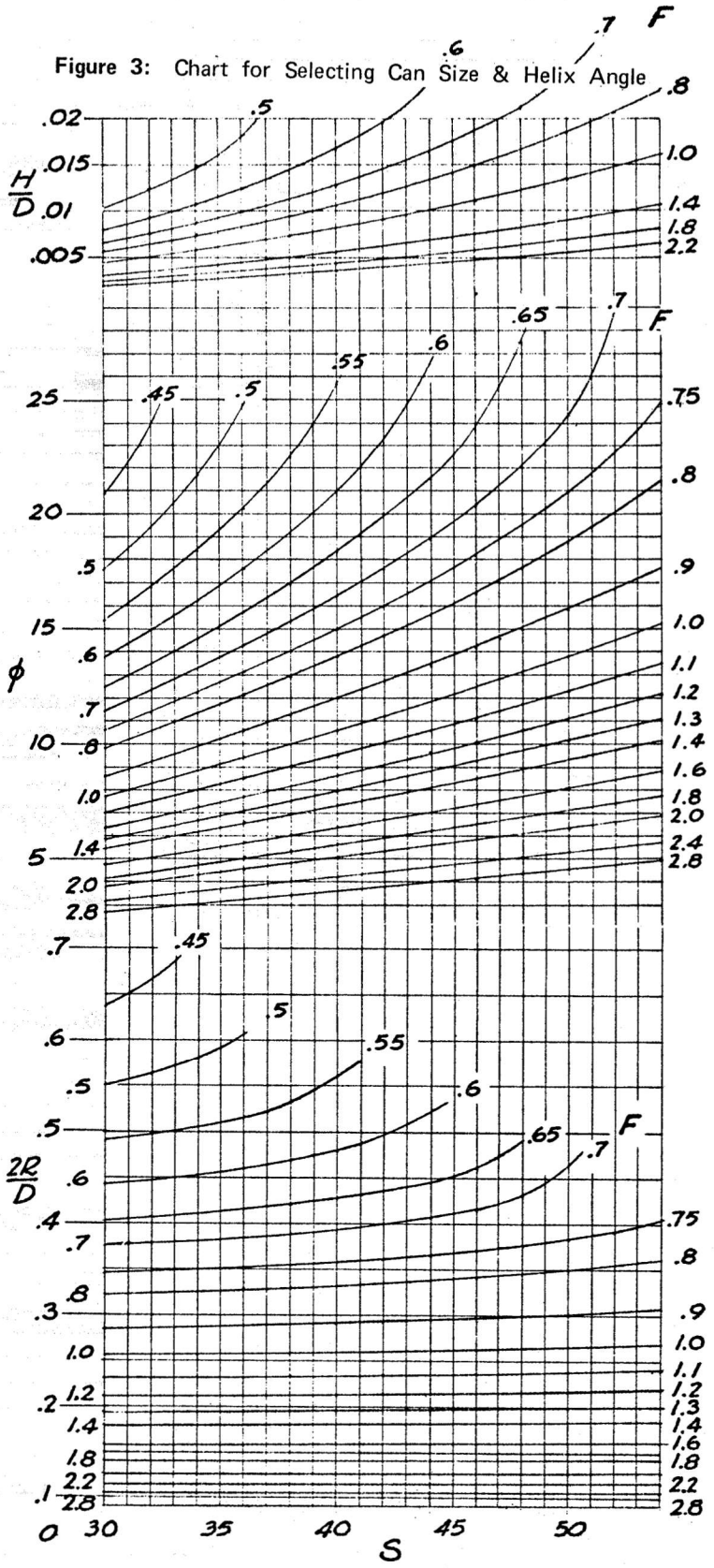
We'll use a blade that starts 30% of the way out from the shaft to the tip; $r_{in}/\frac{1}{2}D = 0.3$. For that value and for P/D = 1.47, from Figure 2 we obtain S = 46.1 degrees.

With the value of S and a value of F of 0.9, we enter Figure 3 and obtain 2R/D = 0.297 and a helix angle θ = 14.2 degrees. The can diameter, then, should have a diameter, 2R, of 2R/D x D or 0.297 x 17 = 5 inches, which by a curious coincidence happens to be the diameter of a large coffee can.

And by another curious coincidence, this prop is awfully similar to the one used on the Pennyplane which Clarence Mather flew for 13 min. 35 sec. (Model Builder, Dec. 72), the best time achieved with a Pennyplane to date.

The can size is not terribly critical. If you have a can that is within 10% of the right size, go ahead and use

Figure 3: Chart for Selecting Can Size & Helix Angle



it — the blade angle error will increase by a degree, but that probably has little effect on the performance. If your favorite can is more than 10% different from the correct diameter, you can change the camber or the pitch or both until you zero in on the can you have.

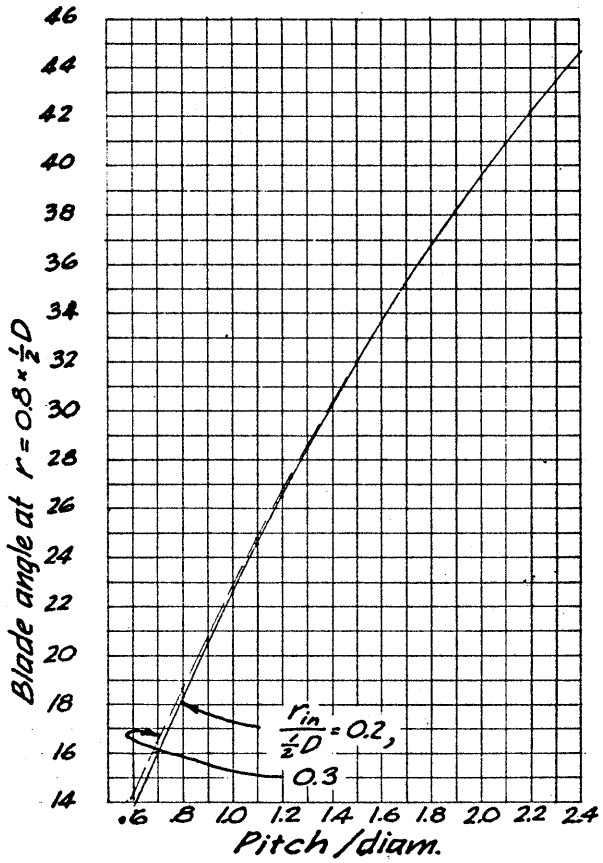


Figure 4: Blade Angle at 80% Radius

APPENDIX

NOMENCLATURE

P	prop pitch
D	prop diameter
a	prop blade angle
c	prop blade chord
h	camber height
L	blade length
R	"cam" radius
r	dist. from propshaft to any point on blade
s	twist rate of blade
ϕ	helix angle; the angle at which the prop blank is wrapped around the can.
H	blade arch height, see figure in appendix
P/D	pitch-diameter ratio
h/c	camber ratio
c/D	chord-diameter ratio
2R/D	ratio of can diameter to prop diameter
F	(h/c)/(c/D)
S ½sD	½sD = twist parameter; total twist for a blade that extends to the propshaft.
H/D	blade arch parameter; a measure of the grotesqueness of the prop.

TWIST RATE REQUIRED, AND BLADE ANGLE SETTING

For a uniform-pitch prop the blade angle αa, at any radius r, is:

$$\alpha_a = \arctan(P / 2\pi r) \qquad (1)$$

where P, the pitch, is a constant.

For a cylindrical prop, the blade angle, αb, varies linearly with the radius:

$$\alpha_b = \alpha_{b,t} + s(\tfrac{1}{2}D - r) \qquad (2)$$

where $\alpha b,t$ is the value of αb at the tip where r = ½D, and s is the rate at which the blade angle changes; a constant.

We define the blade angle error, Δ, by

$$\Delta = \alpha_b - \alpha_a \qquad (3)$$

Upon substituting the values of αa and αb from Eq. 1 and 2 we obtain:

$$\Delta = \alpha_{b,t} + s(\tfrac{1}{2}D - r) - \arctan(P / 2\pi r) \qquad (4)$$

The value of r for which Δ has an extreme value r_{mx} is obtained by letting $d\Delta/dr = 0$:

$$r_{mx} = \sqrt{\frac{P}{2\pi s}\left(1 - \frac{Ps}{2\pi}\right)} \qquad (5)$$

Let us temporarily match the blade angles of the two props (make $\alpha a = \alpha b$) at the tip r = ½D and at some other radius $r = r_{in}$. The blade angles at those points, for both props, we call α_{tip} and α_{in}.

The value of s, the twist rate for the cylindrical prop, is

$$s = (\alpha_{a,in} - \alpha_{a,tip})/(\tfrac{1}{2}D - r_{in}) \qquad (6)$$

where, according to Eq. 1

$$\alpha_{a,in} = \arctan(P / 2\pi r_{in}) \qquad (7)$$

$$\alpha_{a,tip} = \arctan(P / \pi D) \qquad (8)$$

The maxinum value of Δ, from Eq. 4 is

$$\Delta'_{mx} = \left| \alpha_{a,tip} + s(\tfrac{1}{2}D - r_{mx}) - \arctan(P / 2\pi r_{mx}) \right| \qquad (9)$$

where r_{mx} is obtained from Eq. 5, using the value of s obtained from Eq. 6.

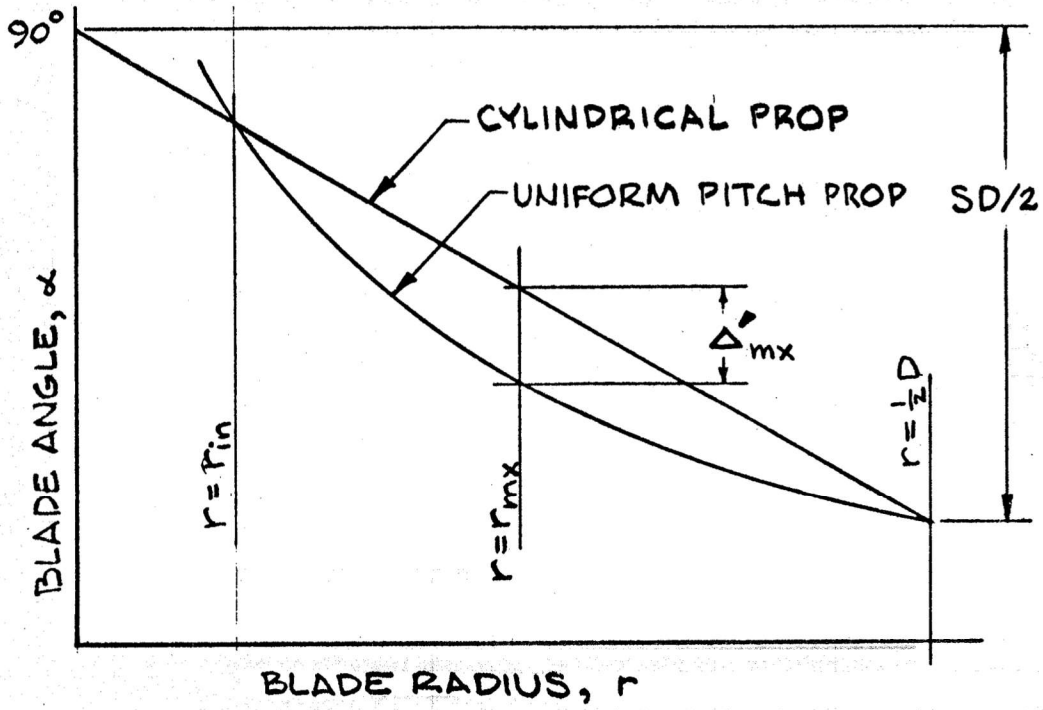


Figure A1

Now let us rotate the blade of the cylindrical prop so as to decrease the blade angle by an amount $\frac{1}{2}\Delta_{mx}$.

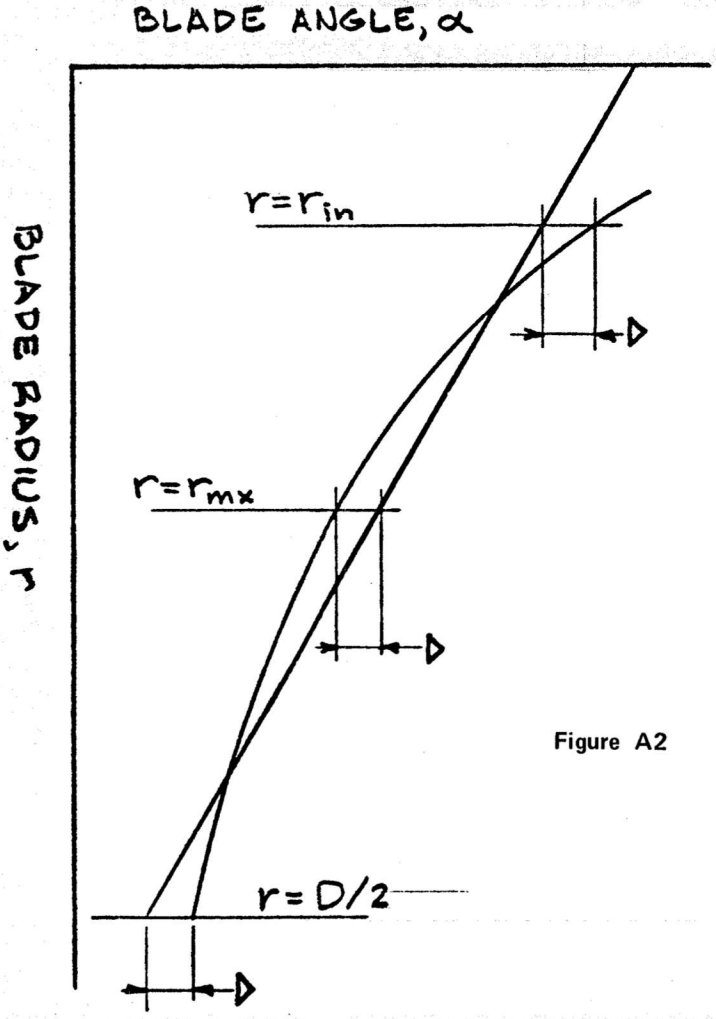


Figure A2

The error Δ will be numerically equal at radius r_{in}, at the tip, and at radius r_{mx}, and will have a value

$$\Delta_{mx} = \tfrac{1}{2}\Delta'_{mx} \qquad (10)$$

The tip blade angle is then

$$\alpha_{a,tip} - \Delta_{mx}$$

so the blade angle at any radius for the cylindrical prop is then, according to Eq. 2

$$\alpha_b = \alpha_{a,tip} - \Delta_{mx} + s(\tfrac{1}{2}D - r) \qquad (11)$$

TWIST RATE

For a very small value of θ the blade twist, $d\alpha$, is equal to θ. For $\phi = 90^\circ$ the blade twist is zero. The following equation has those properties, and it gives the right sort of variation of $d\alpha$ with θ, but beyond that I can offer no defensible proof.

$$d\alpha = \theta \cos \phi \qquad (21)$$

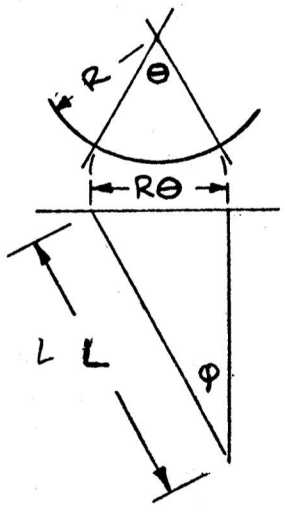


Figure B1

From the geometry of the figure we can write

$$R\,\theta = L \sin\phi \qquad (22)$$

Eliminate θ between Eq. 21 and 22 and solve for the twist rate s, which is just $d\alpha/L$

$$S = d\alpha/L = \sin\phi \cos\phi / R \qquad (23)$$

GENERAL SAGITTA FORMULA

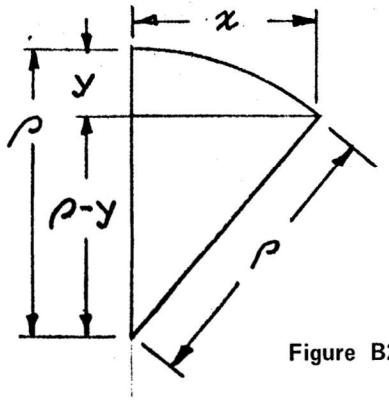


Figure B2

For the triangle

$$\rho^2 = x^2 + (\rho - y)^2$$
$$= x^2 + \rho^2 - 2\rho y + y^2$$

For $y \ll x$, the y^2 term may be neglected, and the equation becomes

$$y = x^2 / 2\rho \qquad (24)$$

CAMBER

Apply the sagitta formula to the above figure:

(Figure B3)

$$h = \frac{(\frac{1}{2}c / \cos\phi)^2}{2R}, \text{ or}$$

$$h = c^2 / (8 R \cos^2\phi) \qquad (25)$$

FORMULA FOR ϕ AND R

Given D, h, c, and s, we would like to be able to calculate R and ϕ. However both R and ϕ appear in Eq. 23 and 25. So we eliminate R between them (solve Eq. 25 for R and substitute into Eq. 23):

$$\sin\phi \cos^3\phi = \frac{s c^2}{8 h} \qquad (26)$$

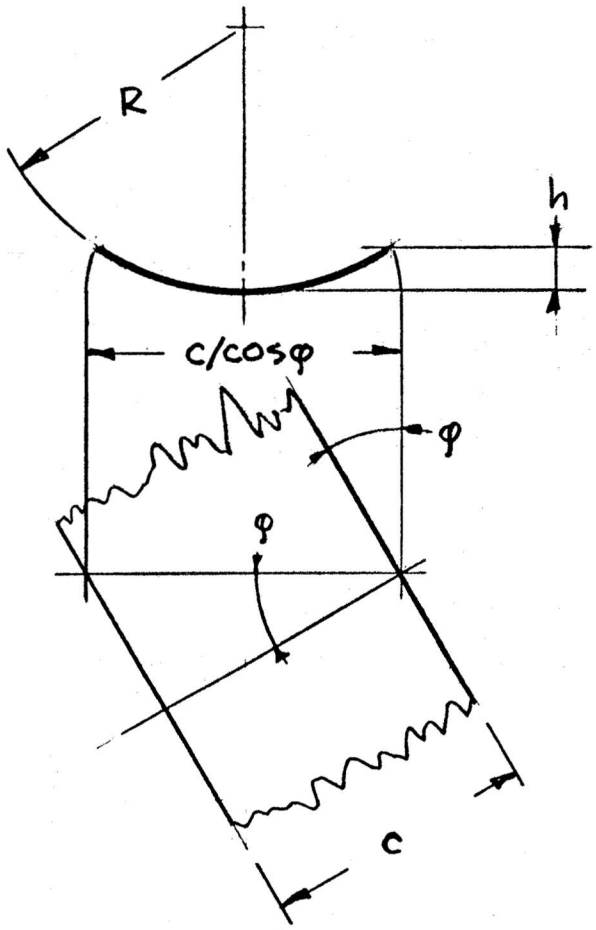


Figure B3

This cannot be solved for ϕ directly, but it can be solved indirectly by Newton's method, or simply by drawing a graph of $sc^2/8h$ vs ϕ.

With ϕ known, R can be obtained directly from Eq. 23:

$$R = \sin\phi \cos\phi / s \qquad (27)$$

The term $sc^2/8h$ in Eq. 26 reaches a maximum value of 0.325 for $\phi = 30^\circ$. Therefore, for too large values of h there are no solutions. For values of the term smaller than 0.325 there are two values of ϕ that satisfy the equation. The smaller value is the proper one; the larger value results in a grotesque prop.

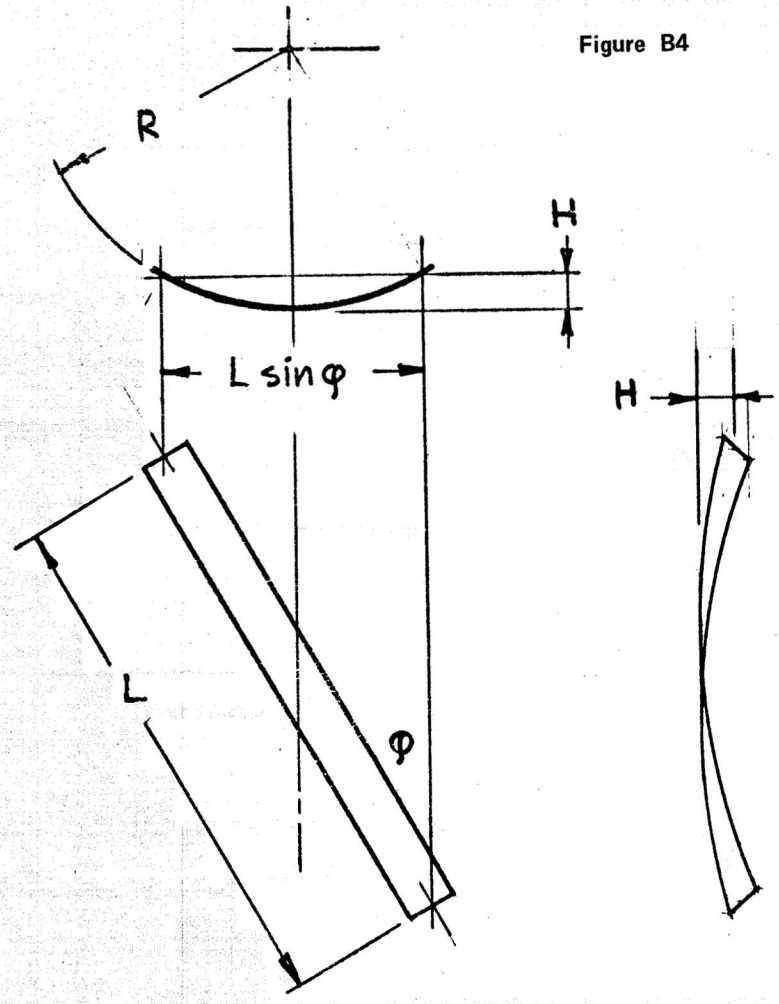


Figure B4

BLADE ARCH

Apply the sagitta formula to the above figure:

$$H = (\tfrac{1}{2} L \sin \phi)^2 / (2R), \text{ or}$$

$$H = L^2 \sin^2 \phi / (8R) \qquad (28)$$

NORMALIZED FORMS

For convenience of calculation and presentation, normalized forms are preferred; that is, we express the camber height h as a fraction of the blade chord c, etc. The following forms are adopted:

Parameter		Normalized Form
Camber height	= h	h/c
Blade width	= c	c/D
Can radius	= R	R/(½D), 2R/D
Blade arch	= H	H/D
Twist rate	= s	sD/2 = S

Equations 26 and 27, in normalized form, become

$$\sin \phi \cos^3 \phi = \frac{(sD/2)(c/D)}{4(h/c)} \qquad (36)$$

$$(2R/D) = \sin \phi \cos \phi / (\tfrac{1}{2} sD) \qquad (37)$$

The blade arch is expressed in terms of a blade that continues all the way to the hub; L = ½D. Eq. 28 then becomes

$$H/D = \frac{\sin^2 \phi}{16(2R/D)} \qquad (38)$$